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## COLUMNS UNDER LOADS OF VARYING ECCENTRICITY

Jim Rhodes<sup>1</sup>

### Summary

A series of tests on small lipped channel section columns is reported in this paper. The columns were loaded with varying degrees of eccentricity. The eccentricity of loading was produced by offsetting the loading line from the neutral axis of the section, and the offset could be set to a different degree at each end of the column. By this means the effect of moment variation along the column could be examined experimentally and the results compared with the predictions of different design codes for this type of loading. The predictions of the 1986 AISI Specification, the British Standard and the Eurocode were compared with the experimental results, and all of these would appear to be quite accurate on the whole in relation to the experimental results.

### Introduction

Part 1.3 of Eurocode 3 (1) was released as a European Prestandard in 1996. This part gives design rules for cold formed thin gauge members and sheeting. The new design specification is at present under thorough scrutiny in the member states of the European Community, as it will have a significant effect on cold-formed steel design throughout the community. The design rules in the new specification have been substantially influenced by the AISI Specification (2) and also by various other specifications, both cold formed and hot rolled.

As part of a calibration exercise against the British standard (3) the rules dealing with the interaction of axial loading and bending moment on columns were compared and found to give substantially different results in some cases. The differences seemed to be greatest in cases when the moment varies along the column. The interaction formulae used in the British code were largely taken from those of the AISI specification (2), and if the safety factors etc. are discarded then there is not a great deal of difference in these two codes.

The Eurocode interaction formulae were taken from the corresponding rules in the parent Eurocode 3, Part 1.1 (4), and had been initially set up for hot rolled steel columns.

The effects of column loading with uniform eccentricity have been found to be safely and conservatively estimated by the AISI/UK specification approach, for example (5), but the UK specification has not, to the writer's knowledge, been calibrated against experiments in the case of varying load eccentricity along a column. To check out the differences in the UK code (and the latest AISI specification (6)) in relation to the

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Eurocode for this type of loading it was decided to embark on a short, preliminary test program on eccentrically loaded columns, with different degrees of eccentricity at both ends to induce moment variation along the columns. The test program, and the results and comparisons with the design rules, are the subject of this paper.

### Design Formulae

The interaction formulae used in the different specifications are subject to different conditions with regard to load and safety factors etc. The Eurocode and the British standard, being limit analysis based, calculate load capacities without safety factors, although the Eurocode uses a material factor of 1.1 rather than unity as is the case with the British code. The 1986 AISI specification, on the other hand, has safety factors incorporated. Thus to perform a meaningful comparison of the different codes, the safety factors, load and material factors must be taken into consideration. In the investigation discussed here it was decided that all factors would be taken out of the formulae used, so that all specification formulae would be taken, with the appropriate modifications where required, to be applicable directly to the evaluation of the ultimate strength of the structural member under consideration.

The members tested were chosen to have a cross section which was fully effective at failure, under either compression or bending, so that the effects of local buckling could be eliminated, and the cross sections assumed to be fully effective. In such a case there may be considered to be no neutral axis shift due to local buckling, and if the member is bent about one axis only, and compressed axially, the interaction formula to be satisfied may be written as follows:-

#### EC3: Part 1.3

The interaction equation in this code may be written, for the columns investigated:-

$$(1) \quad \frac{P}{\chi P_{CS}} + \frac{\kappa M_1}{M_C} \leq 1$$

where  $P$  is the applied axial load,  $M_1$  is the maximum applied moment,  $P_{CS}$  is the stub column capacity,  $M_C$  is the moment capacity in the absence of axial loads,  $\chi$  is a reduction factor for overall buckling such that  $\chi P_{CS} = P_C$ , where  $P_C$  is the column capacity in the absence of applied moments, and the factor  $\kappa$  is obtained as follows:-

$$\kappa = 1 - \frac{\mu P}{\chi P_{CS}} \quad \text{but } \kappa \leq 1.5 \quad (2)$$

In the above expression, the factor  $\mu$  is dependent on the column slenderness and the moment distribution along the column. This factor may be obtained from the following expression:-

$$\mu = \sqrt{\frac{P_{CS}}{P_E}} (2\beta - 4) \quad \text{but} \quad \mu \leq 0.9 \quad (3)$$

Here  $P_E$  is the Euler buckling load, equal to  $\pi^2 EI / L^2$ , and  $\beta$  is an equivalent uniform moment factor. In the case of a moment which varies linearly from  $M_1$  at one end to  $M_2$  at the other, with  $M_1$  having the larger absolute value, as shown in Figure 1, then:-



Figure 1. Primary moment variation along a column

$$\beta = 1.8 - 0.7 \frac{M_2}{M_1} \quad (4)$$

Note that  $M_2$  may be positive or negative, and that the factor  $\mu$  may also be negative, and is indeed negative in all cases where single curvature exists along the column.

#### AISI (1996)

In this specification modifications to the flexural buckling load capacity have been introduced in relation to previous versions of the code, and the interaction equations applying to the problem under investigation, with safety factors discarded, are:-

$$\frac{P}{P_C} + \frac{C_m M_1}{M_C(1 - P/P_E)} \leq 1 \quad (5)$$

where

$$C_m = 0.6 + 0.4 M_2 / M_1 \quad (6)$$

and

$$\frac{P}{P_{CS}} + \frac{M_1}{M_C} \leq 1 \quad (7)$$

### BS 5950: Part 5

The interaction equations used in this specification are based to a large extent on those of the AISI specification, but with some minor differences. Here:-

$$\frac{P}{P_C} + \frac{M_1}{C_b M_C (1 - P/P_E)} \leq 1 \quad (8)$$

where

$$C_b = 1.75 - 1.05 \frac{M_2}{M_1} + 0.3 \left[ \frac{M_2}{M_1} \right]^2 \quad \text{but} \quad C_b \leq 2.3 \quad (9)$$

and

$$\frac{P}{P_{CS}} + \frac{M_1}{M_C} \leq 1 \quad (7)$$

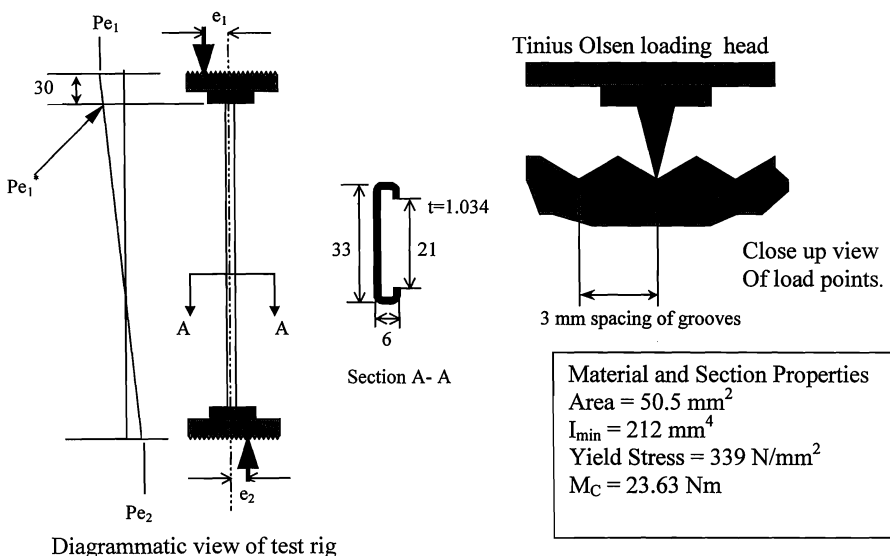
It should be mentioned here that the evaluation of  $P_C$ ,  $M_C$  and  $P_{CS}$  varies from one code to another. In consideration of the specimens tested it was assumed that the moment capacities evaluated by each code were the same, although there are actually small differences, and the column capacities were determined on the basis of the individual specifications. In the case of the moment capacities, as all codes allow elasto-plastic behaviour for the cross section considered the elasto-plastic capacity used in the British code was taken for all three specifications.

### Experimental Investigation

Thirty tests to failure were carried out on small channel cross section specimens under eccentrically applied loading. The specimens were of length varying in increments of 100 mm from 100 mm to 500 mm. The two end blocks through which loading was applied added another 25 mm to the overall length, and therefore the overall length between load points varied from 125 mm to 525 mm. The radius of gyration of the specimens with respect to the minor axis was 2.05 mm, so that the slenderness ratios under examination varied from 61 to 256.

Special clamping blocks were made up to fit the ends of the specimens. The specimens were fitted into the blocks which were then tightened by bolts to fix the ends securely into the blocks. The clamping blocks were bolted to loading blocks which had serrated outer edges, with serrations at 3mm pitch as shown in Figure 2. The loading was applied through knife edged vee blocks as shown in the figure. By selection of appropriate serrations the degree of eccentricity of load at each end of the column could be specified. In practice during tests the column arrangement was aligned so that the loading points were a close to being in the same vertical line as possible, but this was not essential to produce the specified load variation, as

transverse loading was automatically induced at the load points to ensure equilibrium by producing the moment variation specified by the end eccentricities. The tests were carried out in a Tinius Olsen electro-mechanical testing machine.



**Figure 2. Test details**

Six different loading conditions were examined. These were to some extent dictated by the layout of the loading blocks and are as tabulated below:-

Loading Condition	$e_1$ (mm)	$e_2$ (mm)
1	3	3
2	9	-3
3	15	3
4	15	15
5	15	-9
6	9	9

Under each loading condition five column tests were carried out, with overall lengths of 125, 225, 325, 425 and 525 mm. The eccentricity  $e_1$  considered in examination of the tests was modified to take account of the fact that under varying moment the maximum moment actually suffered by the column was  $Pe_1^*$  as shown in Figure 2, due to the fixity within the clamping blocks. The modified eccentricity can be obtained simply from the geometry of the moment diagram as:-

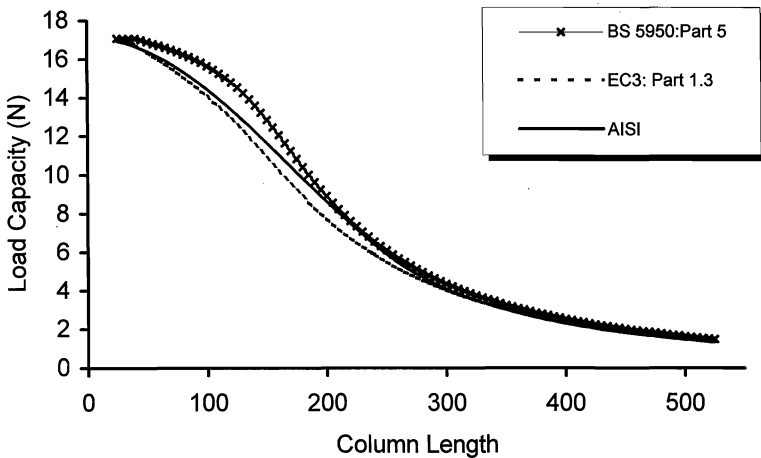
$$e_1^* = \frac{e_1 \times (L - 30) + e_2 \times 30}{L} \quad (10)$$

## Test Results

The failure loads obtained from the tests are tabulated below:-

Loading Condition	125 mm Length	225 mm Length	325 mm Length	425 mm Length	525 mm Length
1	3.83 kN	2.73 kN	1.76 kN	1.34 kN	0.94 kN
2	3.24 kN	2.66 kN	2.23 kN	1.50 kN	1.08 kN
3	1.94 kN	1.50 kN	1.15 kN	0.86 kN	0.71 kN
4	1.50 kN	1.16 kN	0.91 kN	0.71 kN	0.61 kN
5	2.94 kN	2.01 kN	1.65 kN	1.22 kN	0.94 kN
6	1.83 kN	1.66 kN	1.27 kN	1.00 kN	0.79 kN

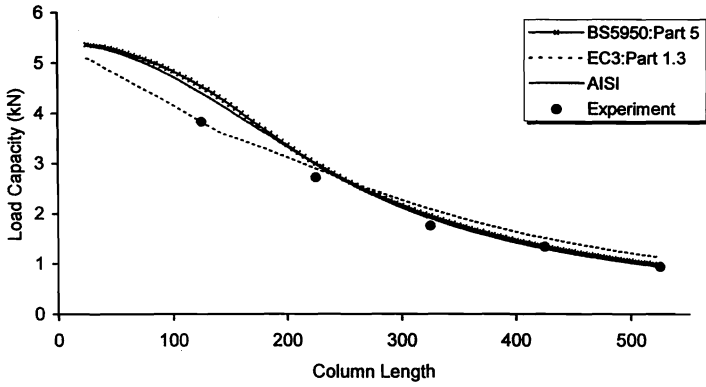
Figure 3 shows the variation in column load with column length predicted for columns under purely axial loading by the three specifications examined. In the case of the Eurocode, the column curve specified for a channel section is curve 'b' with imperfection factor  $\alpha$  of 0.34.



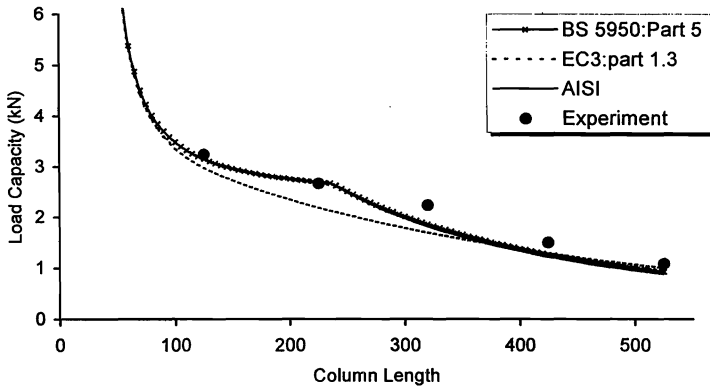
**Figure 3. Column Load Capacity - Uniformly Compressed Columns**

As can be observed the load capacities, particularly for short columns, are substantially in excess of the experimental values obtained under eccentric loading. This indicates that the eccentricities of loading applied have had a substantial effect on the column capacities.

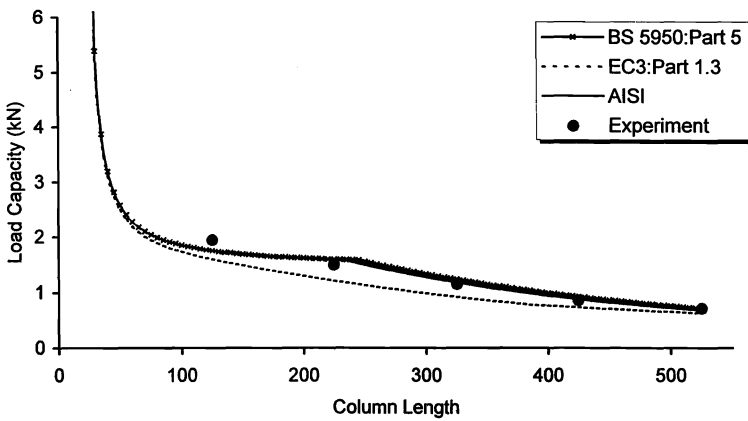
Figures 4 to 9 show the variation of column capacity with variation in column length for each of the 6 loading conditions examined experimentally.



**Figure 4. Column load Capacity - Loading Condition 1**

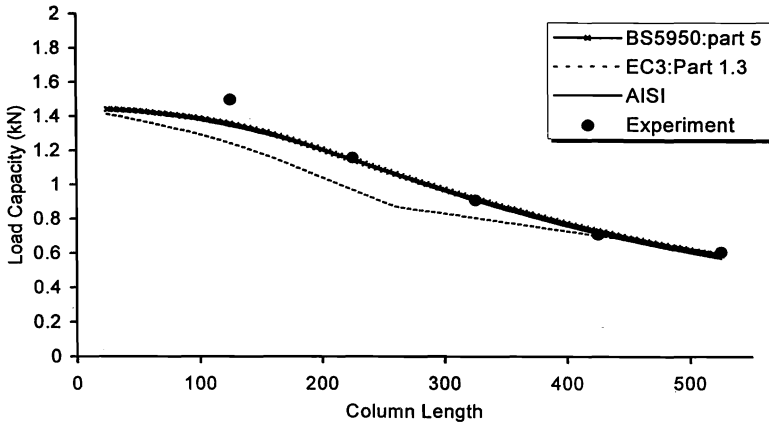


**Figure 5. Column Load Capacity - Loading Condition 2**

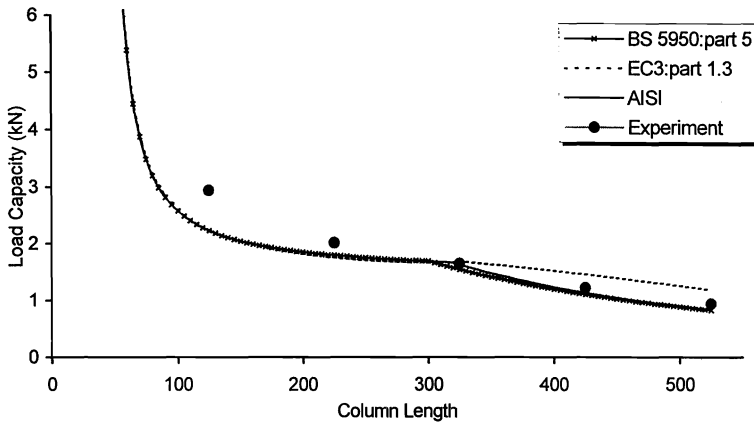


**Figure 6. Column Capacity - loading Condition 3.**

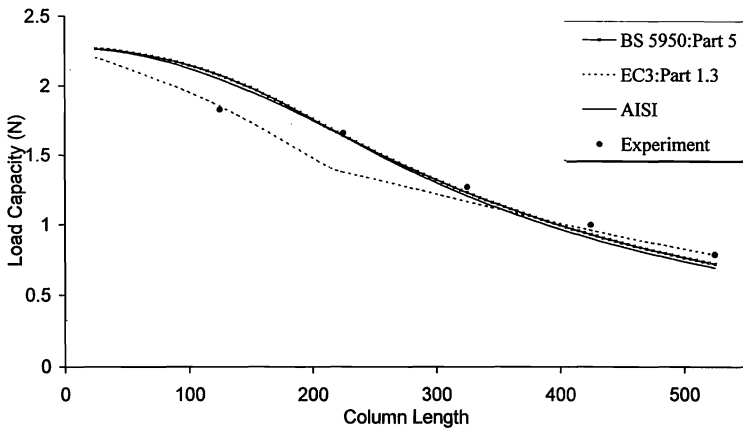




**Figure 7. Column Capacity – Loading Condition 4**



**Figure 8. Column Capacity – Loading Condition 5**



**Figure 9. Column Capacity – Loading Condition 6**

## Observations.

In consideration of the graphs it may be interesting to observe that with a moment capacity of 23.6 Nm the axial load which would be required to cause failure without taking any consideration of the compression effects is, for each different eccentricity:-

$e_1 = 3 \text{ mm}$ ,  $P=7.87 \text{ kN}$

$e_1 = 9 \text{ mm}$ ,  $P=2.62 \text{ kN}$

$e_1 = 15 \text{ mm}$ ,  $P=1.57 \text{ kN}$

In the case of loading conditions which produced constant moment along the column, ie Conditions 1, 4 and 6 the AISI and UK codes produced smooth curves of capacity v length, while the Eurocode curves had a slight kink. For moments which vary substantially along the beam the AISI and UK codes have local strength, specified by interaction equation (7) as the governing criterion for short struts with overall buckling interaction with moment as the dominant effect for longer columns. This is shown by the AISI and BS 5950 curves of Figures 5, 6 and 8 which have fairly sharp changes in slope at mid-range. For these conditions the Eurocode curves are smooth.

Overall the agreement between all three codes and the experimental results is very good. This is to some extent surprising since, particularly for small loading eccentricities, the sensitivity of the results to slight variations in eccentricity is high.

## Conclusions

The brief experimental investigation reported here indicated that the new Eurocode 3:Part 1.3, and the AISI and British codes give realistic evaluations of the load capacities of columns subjected to varying moment along their lengths. This investigation, however, did not go into sufficient depths to provide any comprehensive calibration of the design code predictions, and a substantially more exhaustive investigation would be of aid in promoting assurance as to the safety and accuracy of the relevant formulae.

## Acknowledgements

The experimental results were obtained by Claire Wallace as part of her final year undergraduate research project at the University of Strathclyde, and Miss Wallace's contribution to this work is gratefully acknowledged.

The author should also like to gratefully acknowledge Professor Wei-Wen Yu's assistance in pointing out the new AISI column rules introduced in 1996.

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